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Inverse kinematics Analysis and Solution of Six Degree of Freedom Industrial Robot Based on Projective Method of Descriptive Geometry

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Abstract: *The industrial robot with six degrees of freedom belongs to active mechanical device, it is a complex automatic control system with redundancy, multi variables and essential nonlinearity, the inverse kinematics solution is complex and not unique, No valid closed solutions can be obtained. Based on this, in order to obtain the closed solution of the inverse position of the robot body for experiments, Under the premise that the robot ontology satisfies the Piepre criterion, This paper presents an inverse kinematics analysis and solution method of six degree of freedom industrial robot based on projective method of descriptive geometry. In order to avoid singularity pose, In this paper, the six axes of robot body are divided into two kinds, the first three axes are inverse kinematics analysis, and the descriptive geometry projection method is adopted, by reducing the dimension of the space problem, turn the multiple member mechanism space problem into a plane problem, thus simplifying the calculation process; The latter three axes are solved by conformal geometry algebraic method or algebraic iterative method. Generally speaking, the resulting inverse solutions have multiple sets, the optimal solution can be selected according to the minimum motion range and the minimum motion distance of the joint angle. This method has the advantages of small amount of computation, strong geometry and good real-time performance. In this paper, a series of six degree of freedom industrial robots are used for verification, and the results show that the method is accurate and effective.*

Keywords: *Descriptive geometry projection method, conformal geometry algebra method, six degree of freedom industrial robot, kinematics inverse problem, robot toolbox*

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I. Introduction

Kinematics analysis of robot body is the basis of robot trajectory planning and motion control, Among them, the kinematics analysis of robot body is the force that ignores the motion of the robot body, The constraint relation of attitude, position and speed of robot end effector based on reference coordinate system is studied, Thus derive, Kinematics of robot body is a function of joint position and velocity, Further speak, The key problem of kinematic analysis of robot body is to establish the constraint relationship between the joint variables of robot and the position and attitude of the end effector.

The kinematics analysis of robot body mainly consists of two basic problems, For a given robot ontology, on the basis of known geometric parameters of the member and joint variables, Solve the position and attitude of an end effector relative to a given reference coordinate system, Among them, the given reference coordinate system is usually the Descartes coordinate system fixed on the ground, which is regarded as the overall reference coordinate system of the robot body. This kind of problem is generally called the positive problem of the robot ontology kinematics. The forward kinematics problem of robot body is relatively simple, and it is usually solved by geometric method. Corresponding to the forward kinematics problem of robot body, for a given robot ontology, on the basis of the geometric parameters of the robot body member bar, the position and posture of a given end effector relative to the overall reference coordinate system, and solve definite value of the joint variable, such problems are generally referred to as inverse kinematics problems of robot ontologies. For the six DOF robot ontology, the inverse kinematics problem is complicated, since the equations are usually nonlinear, therefore, it is not always to obtain closed solutions, Moreover, there may be multiple solutions, infinite solutions, and no feasible solutions, but on the basis of applying D-H rule to establish kinematics equation, if the robot ontology satisfies the Piepre criterion(The three adjacent axes of the robot are at one point or the three axes are parallel, experimental results show that the experimental robot Ontology satisfies the Piepre criterion), Closed solutions can be obtained. Because there is coupling between each axis of the robot ontology, the correlation angle cannot be directly obtained from the transformation matrix, in order to avoid the shaft coupling, the general method is algebraic method. this method is clear and easy to understand, but the steps are complex and the real-time is poor. In recent years, in order to replace the traditional algebraic method, many scholars at home and abroad have done a lot of research. North China University of Technology Huang Xiguang [1-4] proposed to solve the robot kinematics analysis of the inverse problem using the method of conformal geometric algebra solution; Shanghai University's Qian Donghai[5] et al, an explicit solution based on screw theory is proposed to solve the inverse kinematics problem of robot body, Harbin Institute of Technology's Yang Haitao[6] proposes a hybrid approach, the joint angle of the first three joints of robot body is solved by geometric projection method, the joint angles of the posterior three joints are solved by algebraic method. These algorithms have improved the efficiency and quality of the inverse solution of robot ontology from different angles and different degrees, Among them, conformal geometry algebra method is more intuitive, and it reveals the geometric relationship between the geometry structure of robot and the motion of robot; The method based on screw theory can replace the D-H rule in low degree of freedom, which can simplify the kinematics analysis of robot body; The method of combining geometric projection and algebraic method is more intuitive, simple in calculation and better in real time, but it is difficult to avoid and deal with singular position.

Based on the above introduction, the problem of inverse kinematics of the robot body is solved by the combination of projective method of descriptive geometry and conformal geometric algebraic method. According to the actual situation, the joint angles of the first three joints of robot body are solved by descriptive geometry projection method; the joint angles of the latter three joints are solved by conformal geometric algebra. The hybrid method has good real-time and geometric visualization [7].

II. The establishment of robot ontology kinematics model

Kinematics analysis of robot body should be carried out, the kinematic model of robot body is established, and then, it is necessary to deduce the kinematics equation of robot body. In order to simplify the modeling process and improve the modeling efficiency, the subject of the experiment is abstracted rationally by using the robot ontology, considering the constraint relationship between the robot body, the link and the trajectory of the robot, the essence of the problem is to determine the coordinate system of the two continuous bars and realize the coordinate transformation between the two member bar. In general, the coordinate system that is attached to the rod is arbitrarily chosen, But for system and general description system describe the relative position and direction of two continuous member bar, In this paper, a general method to solve the kinematic modeling of robot body is put forward by Denavit and Hartenberg in 1955, that is, the famous D-H parameter method. The essence of D-H parameter method is a kind of matrix algebra method. In order to describe the constraint relationship between the bar itself and each other, a coordinate system is established on each link by successively recursive. The experimental robot body belongs to series type, each joint is a rotary joint, and the sketch map of robot body kinematics coordinate system is shown in figure 1.The coordinate system on the fixed base of the robot body is defined as the reference coordinate system, denoted as $O_0(x_0, y_0, z_0)$, the coordinate system on each rod is the reference coordinate system, and the positive direction is determined by the right hand rule, they are denoted as coordinate system {1},{2},{3},{4},{5},{6},the tool coordinate system is denoted as {T}.The following principles need to be followed in the adoption of the D-H parameter method:

(1) Select the axis i along the axis of the joint z_i。

(2) The origin o_i should be located at the intersection of z_i and the common vertical of the axes z_{i-1} and z_i, and so on; the origin o_{i-1} should be at the intersection of the normal vertical line and the axis z_{i-1}.

(3) Select the axis x_i along the normal axis of the shaft z_{i-1} and z_i, pointing $i+1$ from the joint i.

(4) Select axis y_i to form the right hand system.

Each member bar of the robot body is to be completely described, Usually two classes of four parameters are required, the first kind of rod length a_i and rod angle α_i is used to describe the keep rod joint axis relative pendulous, the member bar length a_i is defined as the length of the common axis line i and $i+1$ of the joint axis, the bar angle α_i is defined as the angle between the axis of the plane joint i and $i+1$ perpendicular to the a_i. The second member bar offset d_i and joint angle θ_i are used to describe the relationship between the bars in the multi end linkage of a robot body, The member bar offset d_i is defined as the intersection of two normal line a_i and a_{i-1} in the i axis of the distance, the joint angle θ_i is defined as the angle between a_i and a_{i-1}. In addition, for rotational joints, θ_i is a joint variable, member bar length a_i and member bar corner α_i, member bar offset d_i are constant; For mobile joints, d_i is joint variable, member bar length a_i and member bar corner α_i, Joint angle θ_i, the value remains constant. It has been pointed out above, each joint of the experimental robot Ontology is a rotary joint, and therefore, the core of inverse kinematics analysis of robot body is to solve the joint angle θ_i.

According to the D-H parameter method, after establishing the fixed coordinate system on each member, the coordinate system {*i-1*} and {*i*}, and the relative positional relationship between the adjacent two member *i-1* and *i* can be established by two rotation matrices and two translation matrices[8].

(1) The first step is the rotation transformation of the shaft: The coordinate system {*i-1*} rotates θ_i degrees around the z_{i-1} axis, make the x_{i-1} axis parallel or coplanar with the x_i axis. The axis rotation matrix is defined as A_a, there are:

$$A_a = Rot(z,\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

(2) The second step is translation transformation of axes: The coordinate system {*i-1*} travels d_i around the z_{i-1} axis, make the x_{i-1} axis coincide with the x_i axis or collinear. The axis translation matrix is defined as A_b, there are:

$$A_b = Trans(0,0,d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2)

(3) The third step is the translation transformation of the origin: The coordinate system {*i-1*} moves along the x_i axis translation a_i, the origin of the coordinate system {*i-1*} of *i-1* member bar translation to coincide with the origin of the coordinate system {*i*}. Define the origin and the translation matrix is A_c, there are:

$$A_c = Trans(a_i,0,0) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

(4) The fourth step is collinear rotation transformation: The coordinate system {*i-1*} moves along the x_i axis translation α_i, make the coordinate system {*i-1*} correspond to the coordinate system {*i*}, the axis is same or collinear. Define the origin and the translation matrix is A_d, there are:

$$A_d = Rot(x,\alpha_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & \sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The four sub transformations shown above are described in relation to the reference coordinate system, According to the order of axis rotation transformation, axis translation transformation, origin translation transformation and collinear rotation transformation, The formula of the relative position transformation matrix of member bar *i* adjacent to the member bar *i-1* of the robot body can be obtained, marked as A_i, there are:

$$A_i = Rot(z,\theta_i)Trans(0,0,d_i)Trans(a_i,0,0)Rot(x,a_i) \tag{5}$$

Among them, the physical meaning of A_1 is the pose of the first member relative to the reference coordinate system, and so on, the physical meaning of A_2 is the pose and orientation of second bars relative to the first member, the relevant knowledge of linear algebra can be learned, the reference coordinate system can be used to describe the position and orientation of the second member bars, marked as T_2, there are:

$$T_2 = A_1A_2 \tag{6}$$

In like manner, the physical meaning of A_3 is the position and orientation of third member bars relative to second member bars, there are:

$$T_3 = A_1A_2A_3 \tag{7}$$

Further, for experimental robot ontology, T_6 can be used to describe the transformation matrix of the end effector of the robot body, T_6 is a function of joint variable (joint angle) θ_i, The transformation process of the end effector coordinate system relative to the reference coordinate system can be described completely, there are:

$$T_6 = A_1A_2A_3A_4A_5A_6 \tag{8}$$

If the coordinates of the end effector of the robot body are written as {6}, then the relation between the coordinate system {*i-1*} and the rod coordinate system can be denoted as ${}^{i-1}_{6}T$, there are:

$${}^{i-1}_{6}T = A_iA_{i+1}\cdots A_6 \tag{9}$$

Further promotion, a general expression for the transformation matrix between adjacent members of the robot body can be obtained, marked as ${}^{i-1}_{i}T$, there are:

$${}^{i-1}_{i}T = A_i = Rot(z,\theta_i)Trans(0,0,d_i)Trans(a_i,0,0)Rot(x,a_i) \tag{10}$$

Bring A_a、 A_b、 A_c and A_d into formula ${}^{i-1}_{i}T$ and simplify it, there are:

$${}^{i-1}_{i}T = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

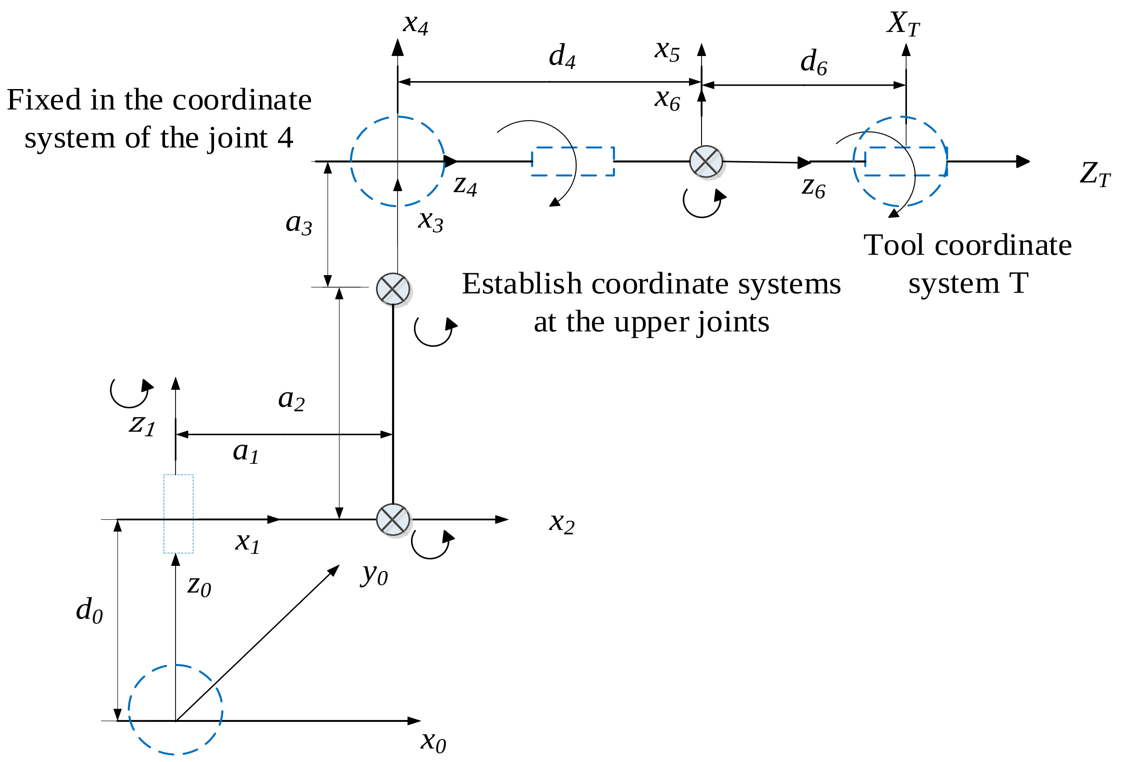


Figure 1: Sketch map of robot body kinematics coordinate system

Based on the above analysis, we can get the D-H parameter table of the experimental robot ontology, which is shown in table 1.

Table1: D-H parameter table of experiment robot ontology

Member bar i	Length of member bar a_i/mm	Member bar corner α_i/°	Member bar offset distance d_i/mm	Joint angle of member bar θ_i/°
0	0	0	d_0	0
1	a_1	-90	0	θ_1
2	a_2	0	0	θ_2-90
3	a_3	-90	0	θ_3
4	0	90	d_4	θ_4
5	0	-90	0	θ_5
6	0	0	d_6	θ_6

According to the D-H parameters of the experimental robot ontology in table 1 and formula 11, The transformation matrix of each member of the robot ontology adjacent to the rod can be found, specific as shown in formula 12.

$${}^{0}_{1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}_{2}T = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1\sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}_{3}^{2}T=\begin{bmatrix} \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ -\cos\theta_2 & \sin\theta_2 & 0 & -a_2\cos\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_{4}^{3}T=\begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3\sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{5}^{4}T=\begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_{6}^{5}T=\begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{T}^{6}T=\begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & cos\,\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12} $$

III. Forward kinematics analysis of robot body

It has already been stated, The direct problem of robot ontology kinematics is about the given robot ontology, On the basis of known geometric parameters and joint variables of the member, The process of solving the pose and attitude of an end effector relative to a given reference coordinate system, Taking formula 12 into equation 8, the kinematics equations of robot ontology can be obtained, there are:

$$ T={}_{1}^{0}T\,{}_{2}^{1}T\,{}_{3}^{2}T\,{}_{4}^{3}T\,{}_{5}^{4}T\,{}_{6}^{5}T\,{}_{T}^{6}T \tag{13} $$

Further, The relative position transformation matrix between adjacent members is brought into formula 13, the solution of the forward kinematics problem of robot ontology is obtained, there are:

$$ T={}_{1}^{0}T\,{}_{2}^{1}T\,{}_{3}^{2}T\,{}_{4}^{3}T\,{}_{5}^{4}T\,{}_{6}^{5}T\,{}_{T}^{6}T=\begin{bmatrix} x_{wx}^{o} & y_{wx}^{o} & z_{wx}^{o} & p_{wx}^{o} \\ x_{wy}^{o} & y_{wy}^{o} & z_{wy}^{o} & p_{wy}^{o} \\ x_{wz}^{o} & y_{wz}^{o} & z_{wz}^{o} & p_{wz}^{o} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14} $$

Among them, The pose vector of the end effector of the robot ontology is expressed as（$\mathbf{x}_{\mathbf{w}}^{\mathbf{o}}$, $\mathbf{y}_{\mathbf{w}}^{\mathbf{o}}$, $\mathbf{z}_{\mathbf{w}}^{\mathbf{o}}$, $\mathbf{p}_{\mathbf{w}}^{\mathbf{o}}$）, by formula 14, it can be concluded that T is a function of θ_i, the position of the end effector is only related to A of functional relationship(i=1,2,3), Independent of the posterior three joints[9]. Thus proves, the position of the end effector is determined by the arm joint of the robot body (the first three joints), The attitude of the end effector is determined by the wrist joint of the robot body (the latter three joints).

IV. Inverse kinematics analysis of robot body based on projective method of descriptive geometry

According to the rotation range of each joint of the robot body, as an explicit constraint, the following steps are given to solve the joint angles of the first three joints of robot ontology by using the projective method of descriptive geometry:

(1) Solving the first joint angle of robot ontology θ_1

The essence of projective method of descriptive geometry is dimension reduction; turn the multiple member mechanism space problems into a plane problem, thus simplifying the calculation process. According to the Paul algebra method, there are:

$$ {}_{1}^{0}T^{-1}T_6={}_{6}^{1}T \tag{15} $$

$$ {}_{0}^{6}T={}_{0}^{T}T({}_{6}^{T}T)^{-1} \tag{16} $$

According to the formula 15 and 16 can be obtained, The joint angle can be solved by inverse transformation matrix; the general form of T is shown in formula 14. As shown in figure 2, The first joint angleθ_1 is projected onto the two-dimensional plane by the projective method of descriptive geometry, It is assumed here that the position coordinates of the robot body end effector coordinate system in the reference coordinate system is P (x_p,y_p,z_p), According to the transformation criteria of the transformation matrix and the inverse solution of the transformation matrix, Further arrange, there are:

$$ \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}=T\begin{bmatrix} 0 \\ 0 \\ -d_6 \\ 1 \end{bmatrix}=\begin{bmatrix} p_x-a_xd_6 \\ p_y-a_yd_6 \\ p_z-a_zd_6 \\ 1 \end{bmatrix} \tag{17} $$

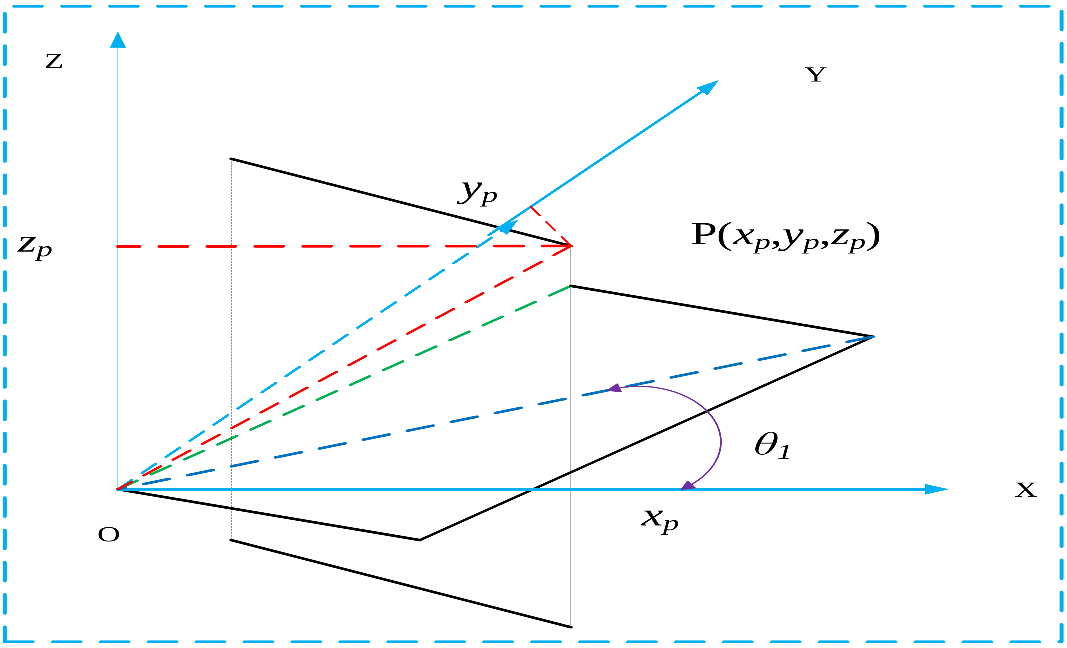


Figure 2: Sketch map of solving the first joint angle

According to the actual situation of the problem, Formula 17 equal value principle on both sides of an equal sign, Because (x_p,y_p) is known, Combining the projective geometric meaning ofθ_1 in figure 2, The first joint angleθ_1 can be obtained, there are:

$$\theta_1 = arctan2(y_p, x_p) = arctan2(p_y - a_y d_6, p_x - a_x d_6)$$
(18)

According to the motion range of robot joints, In experiment, the first joint angle of robot body is θ_1, and the range is ±180 degrees, according to the geometric meaning of projection of θ_1, θ_1 has two complementary solutions, then another solution of θ_1 is:

$$\theta_1 = \pi - arctan2(y_p, x_p) = \pi - arctan2(p_y - a_y d_6, p_x - a_x d_6)$$
(19)

(2) Solving the second joint angle of robot ontology θ_2

Taking into account the two values of the θ_1 solved above are complementary, having similar projective geometric meaning, therefore, θ_2 is considered only when θ_1 is the first solution[10], the schematic diagram of solving the second joint angle θ_2 is shown in figure 3.According to the transformation criteria of the transformation matrix and the inverse solution of the transformation matrix, the relative positional relation between the second joint coordinate systems and the reference coordinate system can be obtained, there are:

$$\begin{bmatrix} {}^2_p x \\ {}^2_p y \\ {}^2_p z \\ 1 \end{bmatrix} = {}^0_1T\,{}^1_2T \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = {}^0_2T \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} \quad (20)$$

Refer to the projective geometric meaning of figure 3, furthermore, a solution of θ_2 can be obtained by further simplification, there are:

$$\theta_2 = arctan2({}^2_p y, {}^2_p x) - \left[\pm arccos\left(\frac{a_2^2 + (x_p - x_2)^2 + (y_p - y_2)^2 + (z_p - z_2)^2 - a_3^2 - d_1^2}{2a_2\sqrt{(x_p - x_2)^2 + (y_p - y_2)^2 + (z_p - z_2)^2}} \right) \right] \quad (21)$$

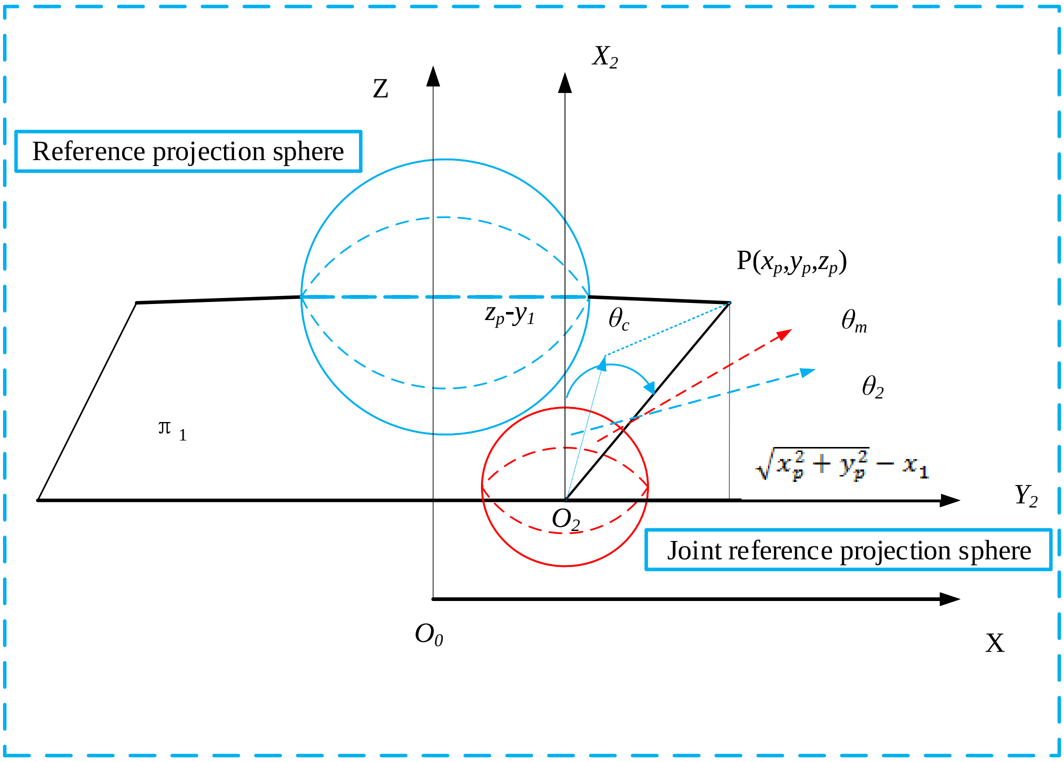


Figure 3: Sketch map of solving the first solution of the second joint angle

By analyzing the geometric meaning of θ_2 and the valid range of θ_2, θ_2 has two values, refer to the projective geometric meaning of figure 4, and further simplify, we can get another solution of θ_2, there are:

$$\theta_2 = arctan2\left({}^2_p y, {}^2_p x\right) + \left[\pm arccos\left(\frac{a_2^2+(x_p-x_2)^2+(y_p-y_2)^2+(z_p-z_2)^2-a_3^2-d_1^2}{2a_2\sqrt{(x_p-x_2)^2+(y_p-y_2)^2+(z_p-z_2)^2}}\right)\right] \tag{22}$$

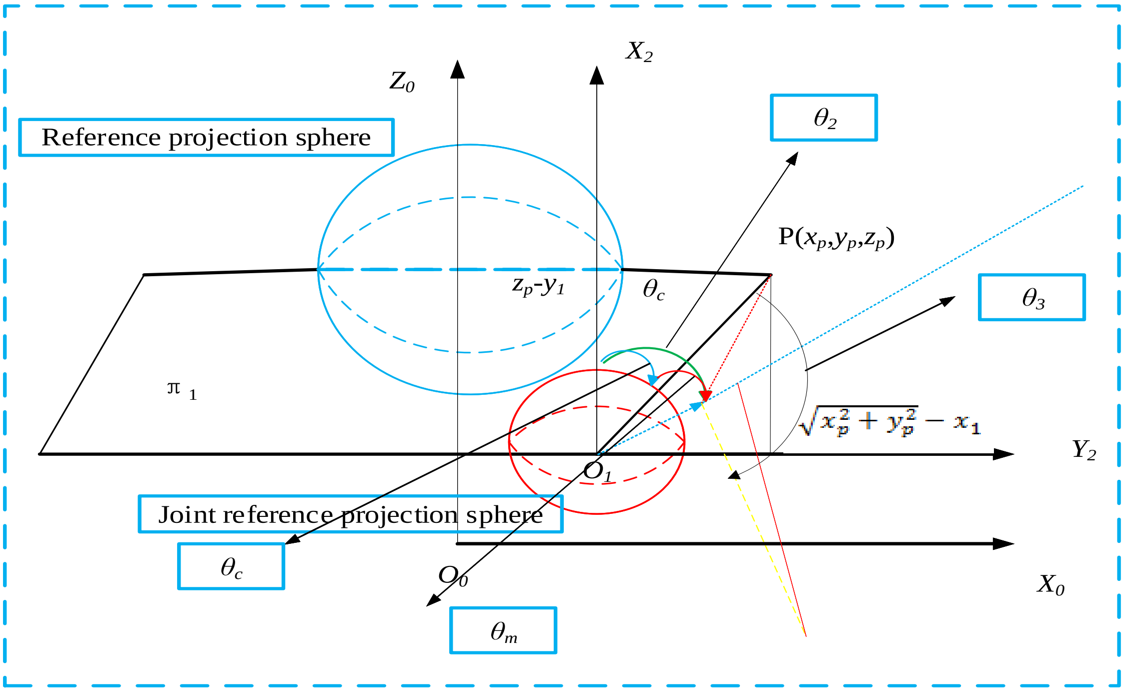


Figure 4: Sketch map of solving the second solution of the second joint angle

(3) Solving the third joint angle of robot ontology θ_3

Solving the third joint angles of robot ontology θ_3 needs to be solved on the basis of θ_1 and θ_2, in order to simplify the solution process, intermediate value method is adopted in this research[11], first solve the intermediate values that are easily represented by known parameters, then the middle value is used to represent the θ_3, according to the actual range of θ_3, refer to the projective geometric meaning of figure 5, two solutions of θ_3 can be obtained by further simplifying, there are:

$$\theta_c = arccos\left(\frac{a_2^2-(x_p-x_2)^2-(y_p-y_2)^2-(z_p-z_2)^2+a_3^2+d_1^2}{2a_2\sqrt{a_3^2+d_1^2}}\right) \quad (23)$$

$$\theta_m = \pi - \arctan\left({}^{d_1}/_{a_3}\right) \quad (24)$$

Further, two solutions of third joint angles of robot body are obtained, there are:

$$\theta_3 = -(\theta_c - \theta_m) = \pi - \arctan\left({}^{d_1}/_{a_3}\right) - arccos\left(\frac{a_2^2-(x_p-x_2)^2-(y_p-y_2)^2-(z_p-z_2)^2+a_3^2+d_1^2}{2a_2\sqrt{a_3^2+d_1^2}}\right) \quad (25)$$

$$\theta_3 = (\theta_c + \theta_m) - 2\pi = arccos\left(\frac{a_2^2-(x_p-x_2)^2-(y_p-y_2)^2-(z_p-z_2)^2+a_3^2+d_1^2}{2a_2\sqrt{a_3^2+d_1^2}}\right) - \pi - \arctan\left({}^{d_1}/_{a_3}\right) \quad (26)$$

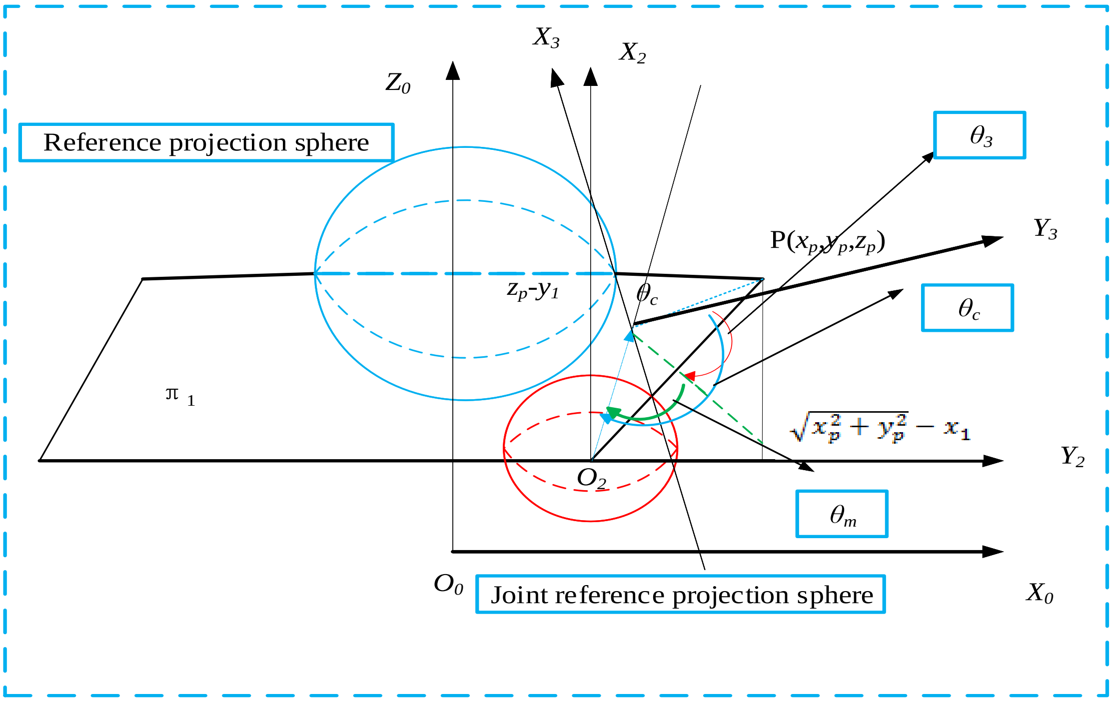


Figure 5: Sketch map of solving the third joint angle

(4) The posterior three joint angles of robot ontology are solved by conformal geometry algebraic method

Conformal Geometric Algebra(CGA) was founded by Li Hongbo researchers in 1997, after a short period of more than ten years of development, it has become the mainstream of international geometric algebra research, especially in computer graphics. The experiment uses a robot ontology as a 6R (R representation rotating pair) series robot, among them, the position of the robot end effector(Expressed as P_t), The posture of the robot end effector(expressed as N_t), The terminal plane of the robot end effector(expressed as π_t), the length of each member (expressed as L_1- L_6) is the known parameter, he required parameters are the three posterior joint angles (expressed as θ_4、θ_5、θ_6), position of the posterior three joints (P_4、P_5、P_6). As shown in figure 6, the position of the three joints is solved first, the three joint angles were then solved[12]. As you can see from figure 6, Joint P_1- P_5 coplanar, the plane is written as π_4, according to conformal geometry algebra correlation theory, there are:

$$\pi_4^* = P_1 \wedge P_2 \wedge P_5 \wedge e_\infty \quad (27)$$

The line is ${}_{P_5}^{P_t}L^*$ that suppose the joint P_5 is connected with position of end effector of robot ontology P_t, there are:

$$ {}^{P_t}_{P_5}L^* = P_5 \wedge P_t \wedge e_\infty \tag{28}$$

The projection of line ${}^{P_t}_{P_5}L^*$ on plane π_4 is expressed as:

$$L^*_{\text{投}} = \frac{\pi_4^* \cdot {}^{P_t}_{P_5}L^*}{\pi_4^*} \tag{29}$$

According to the expression of geometric elements of conformal geometric algebra, take joint P_5 as the centre of sphere, L_4 is a radius, and the projected sphere Q_5 constructed can be represented as:

$$Q_5 = P_5 - \frac{1}{2}L_4^2 e_\infty \tag{30}$$

The line L_t intersects the projected sphere Q_5 and generates a series of point pairs, there are:

$$P_{P_4} = Q_5 \wedge L_{\text{投}} \tag{31}$$

According to figure 6, the joint P_4 is a line L_t intersecting the projected sphere Q_5 and generating any point in a series of point pairs, Subject to recent principles[13], select one point as the position of the joint P_4. The calculation procedure of joint P_5 and P_6 is similar to the calculation procedure of joint P_4, the difference is simply that the projection sphere is replaced by the corresponding translational spinor, therefore, the expressions of joint P_5 and P_6 are given, where T_5 is expressed as the translational spinor of the joint P_5, where T_6 is expressed as the translational spinor of the joint P_6, there are:

$$P_5 = T_5 P_t \tilde{T}_5 \tag{32}$$

$$P_6 = T_6 P_t \tilde{T}_6 \tag{33}$$

After solving the positions of joints P_4, P_5, and P_6, Expressions for geometric elements based on known conditions and conformal geometric algebra, the straight line that can be passed through each joint point, there are:

$$\begin{cases} l_4^* = P_4 \wedge P_5 \wedge e_\infty \\ l_5^* = P_5 \wedge P_6 \wedge e_\infty \end{cases} \tag{34}$$

According to figure 6, the joint angle θ_4 can be represented by the angle between the line l_3 and the l_4, in like manner, the joint angle θ_5 can be represented by the angle between the line l_4 and the l_5, there are:

$$\cos\theta_4 = \frac{l_3^* \cdot l_4^*}{|l_3^*||l_4^*|} \tag{35}$$

$$\cos\theta_5 = \frac{l_4^* \cdot l_5^*}{|l_4^*||l_5^*|} \tag{36}$$

According to conformal geometric algebra theory, the sixth joint angle θ_6 affects only the posture of the end effector, θ_6 can be expressed as the angle between the plane of the first 5 joint angles and the plane of the end effector plane, formula 35 and 36 are solved by positional positive solution[14], after a simple collation, you can get θ_6.

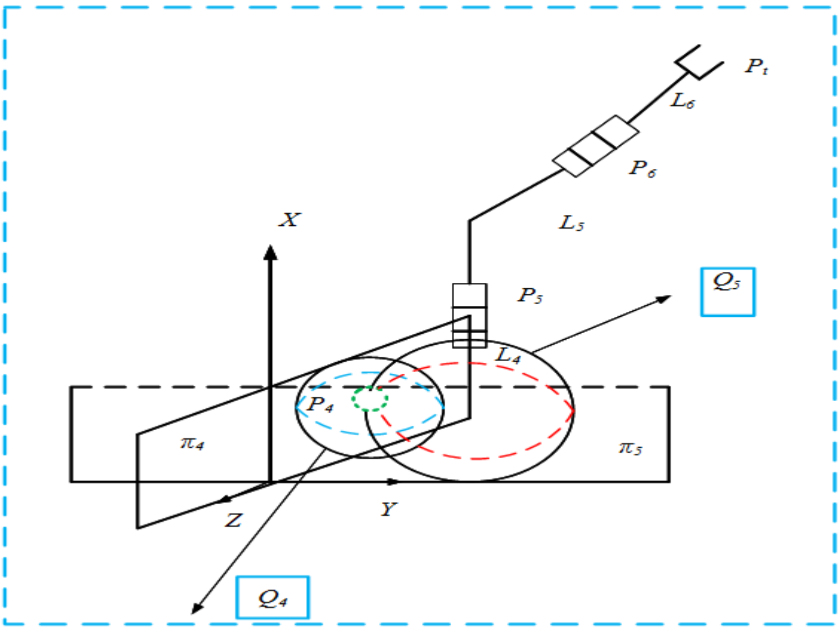


Figure 6: Sketch map of solving the posterior three joint angles

V. Kinematic simulation and verification of robot ontology based on Matlab

In order to verify the validity of the kinematic model and the correctness of the positive and inverse solutions of the robot kinematics, project in Matlab2015b environment, robot toolbox（Robotics Toolbox for Matlab V9.10）the kinematics of robot ontology is simulated and validated. With Matlab GUI toolbox, the graphic control interface of robot ontology kinematics simulation is developed, the working principle of the robot toolbox is relatively simple, In solving the positive problem, each joint of the robot ontology is constructed first, then combine the joints, and finally, the robot ontology model is built. Under the premise of D-H parameters, joint type and quantity, quality and length of member bar of robot ontology, the parametric and quantitative model of robot ontology can be constructed by robot toolbox, then the direct problem is solved; when solving inverse problems, firstly, the Jacobian matrix D-H parameters known to derive the pseudo inverse transpose or determine the iterative direction, then give each joint of the robot body a torque that moves toward the target, finally, the joint variables are converged to the target position by finite iterations[15].

The basic idea of direct kinematic problem simulation is to give the joint angle θ_i of each joint first, then the positive kinematics equations and the robot toolbox are used to solve the position and orientation matrix respectively, determine whether the forward kinematics equation is correct by whether the two pose matrices are equal or not.

After the program runs, Enter p0 and p1 at the command interface, the resulting matrices p0 and p1 are shown in figure 7, using the robot toolbox, a simple structural model of robot ontology corresponding to matrix p0 and p1 is obtained. The working state diagram is shown in figure 8, by comparing the corresponding parameters, we can see that, the results obtained by the two are corresponding to each other, thus the forward kinematics equation is correct.

```
p0 =

    0.0000   -0.0000    1.0000    8.9400
   -0.7071   -0.7071   -0.0000   -4.9497
    0.7071   -0.7071   -0.0000   -3.5497
         0         0         0    1.0000
```

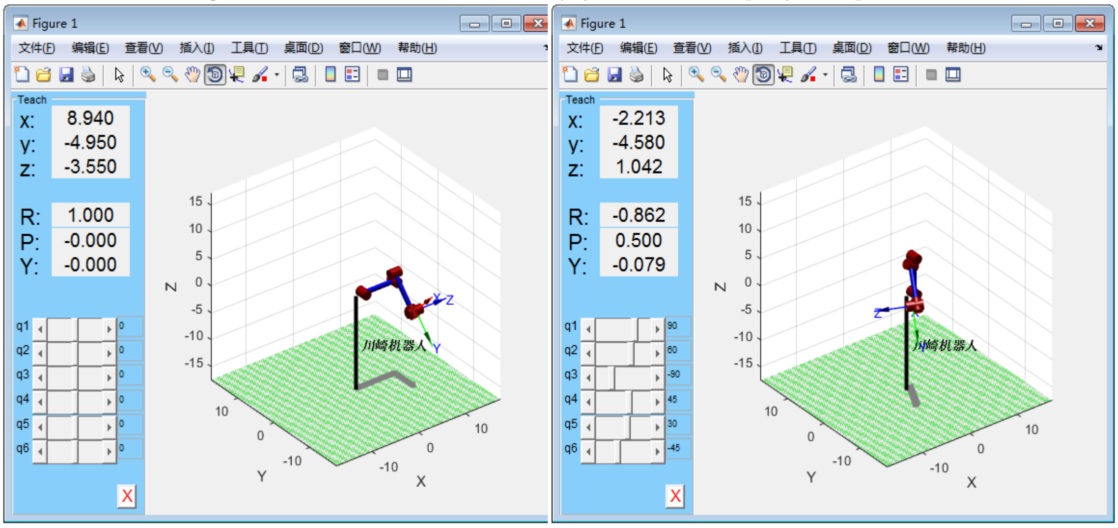
theta=[0 0 0 0 0 0]

```
p1 =

    0.4415   -0.2477   -0.8624   -2.2130
    0.6660   -0.5536    0.5000   -4.5797
   -0.6013   -0.7951   -0.0795    1.0422
         0         0         0    1.0000
```

theta1=[pi/2 pi/3 -pi/2 pi/4 pi/6 -pi/4]

Figure 7: Pose matrix of robot body generated after program operation



theta=[0 0 0 0 0 0]

theta1=[pi/2 pi/3 -pi/2 pi/4 pi/6 -pi/4]

Figure 8: Work state diagram of robot ontology generated after program operation

The kinematic direct problem is verified after completion, Enter kinematics inverse problem simulation interface, the basic idea of kinematic inverse problem simulation verification is to give the pose matrix of the test point (the same as above), Then the joint angle θ_i is solved by inverse kinematics equation and robot toolbox, Determine whether the inverse kinematics equation is correct by whether the joint angles of the two are equal[16].

As shown in figure 9, the joint angles of the test points are obtained, find by contrast, the inverse solution is not exactly the same as the joint angle given by the test point, the reason is that the function of the inverse kinematics is error in the robot toolbox, it can only give a group of solutions, after considering these factors, it can be concluded that the inverse kinematics equation is correct[17].

```
q0 =

     0     0     0     0     0     0

q1 =

    2.2920   -2.0760    0.2302    1.5715    2.4934    2.4934
```

Figure 9: The joint angle of the robot ontology generated after the program runs

VI. Conclusion

Based on descriptive geometry projection and conformal geometric algebra, an inverse kinematics analysis and solution method for hybrid six degree of freedom industrial robots is proposed. By dividing the six axes of robot body into two categories, the algorithm can avoid singular pose, Among them, the inverse kinematics of the first three axes is analyzed by the projective method of descriptive geometry, by reducing the dimension of the space problem, turn the multiple member mechanism space problem into a plane problem, thus simplifying the calculation process; The latter three axes are solved by conformal geometric algebra, it has strong geometric intuition. After solving the inverse set of multiple sets, the optimal solution is selected according to the minimum motion range and the minimum motion distance of the joint angle. The project is verified by a series of six degree of freedom industrial robots, the verification results show that the method is accurate and effective. It has less computational complexity, better geometric intuition and better real-time performance.

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References

[1] Huang Xiguang, Huang Xu, Li Qicai. Inverse displacement solutions of serial robot based on conformal geometric algebra [J]. Journal of Xi'an Jiaotong University, 2017, 51(1):9-12.
[2] Huang Xiguang, Liu Bing, Li Qicai. Inverse kinematics analysis of a 6 dergee-of-freedom articulated robot using conformal geometric algebra [J]. Journal of Shanghai Jiao Tong University, 2016(s1):86-89.
[3] Zhou Dongdong, Wang Guodong, Xiao Juliang, et al. Design and kinematic analysis of a new modular reconfigurable robot [J]. Journal of engineering design, 2016, 23(1):74-81.
[4] Zhang Zhonghai, Li Duanling. A conformal geometric algebra method for kinematic analysis of spatial parallel mechanisms [J]. Journal of agricultural machinery, 2015, 46(4):325-330.
[5] Qian Donghai, Wang Xinfeng, Zhao Wei, et al. Inverse solution algorithm of 6 degree of freedom robot based on screw theory and Paden-Kahan subproblem [J]. Journal of Mechanical Engineering, 2009, 45(9):72-76.
[6] Yang Haitao. Research on arc welding robot control system based on WinCE [D]. Harbin Institute of Technology, 2011.
[7] L.Sardana,M.K.Sutar, P.M.Pathak. A geometric approach for inverse kinematics of a 4-link redundant In-Vivo robot for biopsy[J]. Robotics and Autonomous Systems, 2013, 61(12): 1306-1313.
[8] Pham Q C. A General, Fast, and Robust Implementation of the Time-Optimal Path Parameterization Algorithm[J]. IEEE Transactions on Robotics, 2014, 30(6):1533-1540.
[9] Oberherber M, Gattringer H, Müller A. Successive dynamic programming and subsequent spline optimization for smooth time optimal robot path tracking[J]. 2015, 6(2):245-254.
[10] Ardeshiri T, Norrlöf M, Löfberg J, et al. Convex Optimization approach for Time-Optimal Path Tracking of Robots with Speed Dependent Constraints[J]. IFAC Proceedings Volumes, 2011, 44(1):14648-14653.
[11] Liao Wu,Hongliang Ren. Finding the Kinematic Base Frame of a Robot by Hand-Eye Calibration Using 3D Position Data[J]. IEEE Transactions on Automation Science and Engineering, 2017, 14(1): 314 – 324.
[12] Huang Xiguang, Liu Bing, Li Qicai. An inverse kinematics conformal geometric algebra method for 6 DOF joint robots [J]. Journal of Shanghai Jiao Tong University, 2016(s1):86-89.
[13] Hong Lei, Ji Baojian, Cai Ganghong, et al. Inverse Kinematics Algorithm for Industrial Robot of ABB1410 Based on Screw Theory [J]. Mechanical design and manufacturing, 2016(4):190-193.
[14] Shi Xiaojia, Zhang Fumin, Qu Xinghua, et al. Position and Attitude Measurement and Online Errors Compensation for KUKA Industrial Robots [J]. Journal of Mechanical Engineering, 2017, 53(8).

[15] Zhao Liang, Zhang Yide, Hu Xuxiao. Simulation collision detection algorithm for industrial robot based on mesh envelope [J]. China Mechanical Engineering, 2017, 28(3):316-321.

[16] Li Ningsen, Zhou Bo, Zhang pan, pan, et al. Real-time inverse kinematics algorithm for 6R robots with non-spherical wrist[J]. Journal of Huazhong University of Science and Technology (NATURAL SCIENCE EDITION), 2015(s1):461-467.

[17] Zhang Guoying, Liao Yajun, Liang Feng, et al. A kind of spherical parallel wrist mechanism and its kinematics analysis [J]. Robot, 2017, 39(2):167-175.

Zhenhua Wang. "Inverse kinematics Analysis and Solution of Six Degree of Freedom Industrial Robot Based on Projective Method of Descriptive Geometry." IOSR Journal of Electronics and Communication Engineering (IOSR-JECE) 12.4 (2017): 31-42.